Radio Interferometry: facet-based approach to inverting the RIME

LOFAR school presentation

Etienne Bonnassieux, postdoctoral research fellow at UniBo
In collaboration with:
Tim Shimwell & the ~*constellation of interferometers*~

Outline

1. Mathematical Framework - the RIME

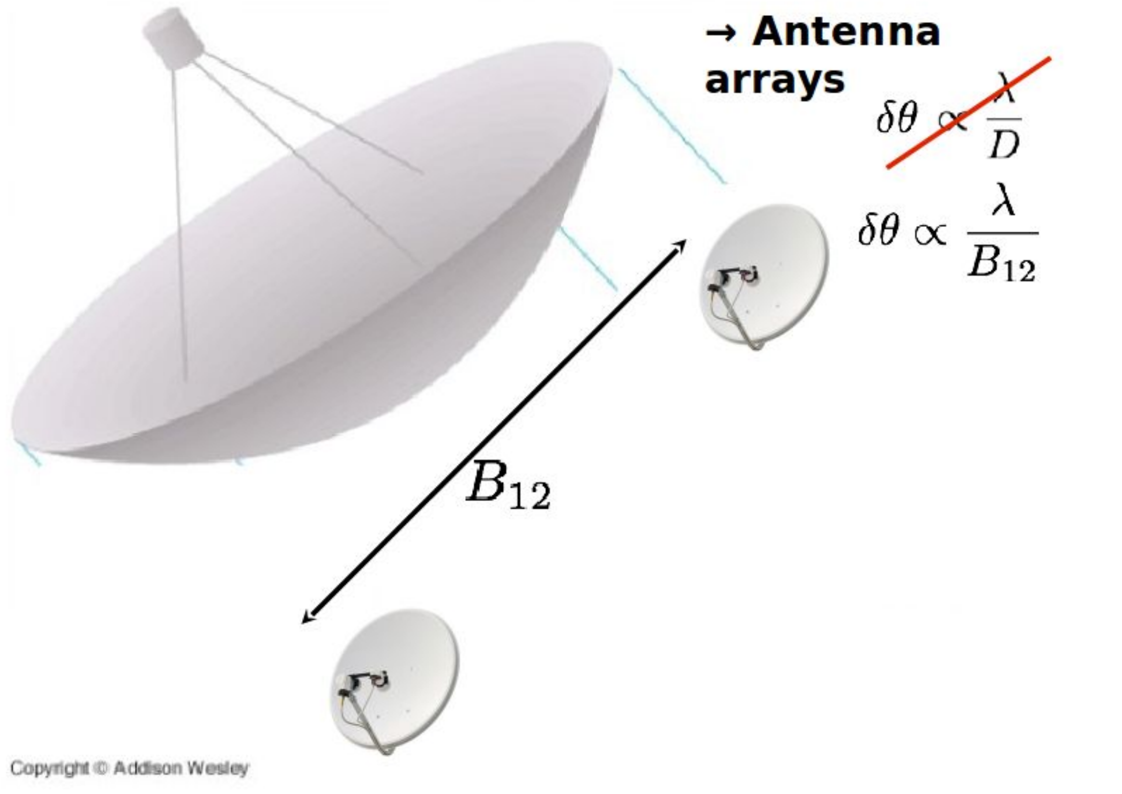
2. The inverse problem of interferometry

3. Facet-based direction-dependent calibration

Radio interferometry

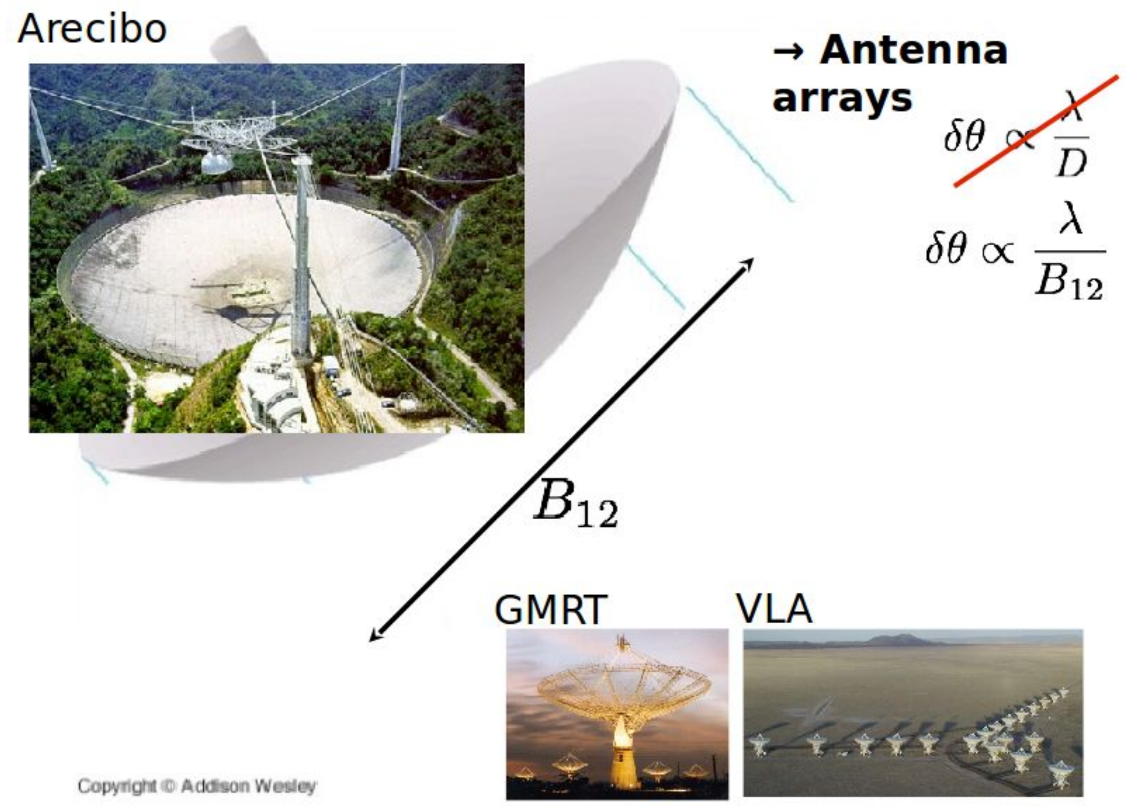
GMRT near Pune, India

Why bother with interferometry?



Slide credit:
Julien Girard

Why bother with interferometry?



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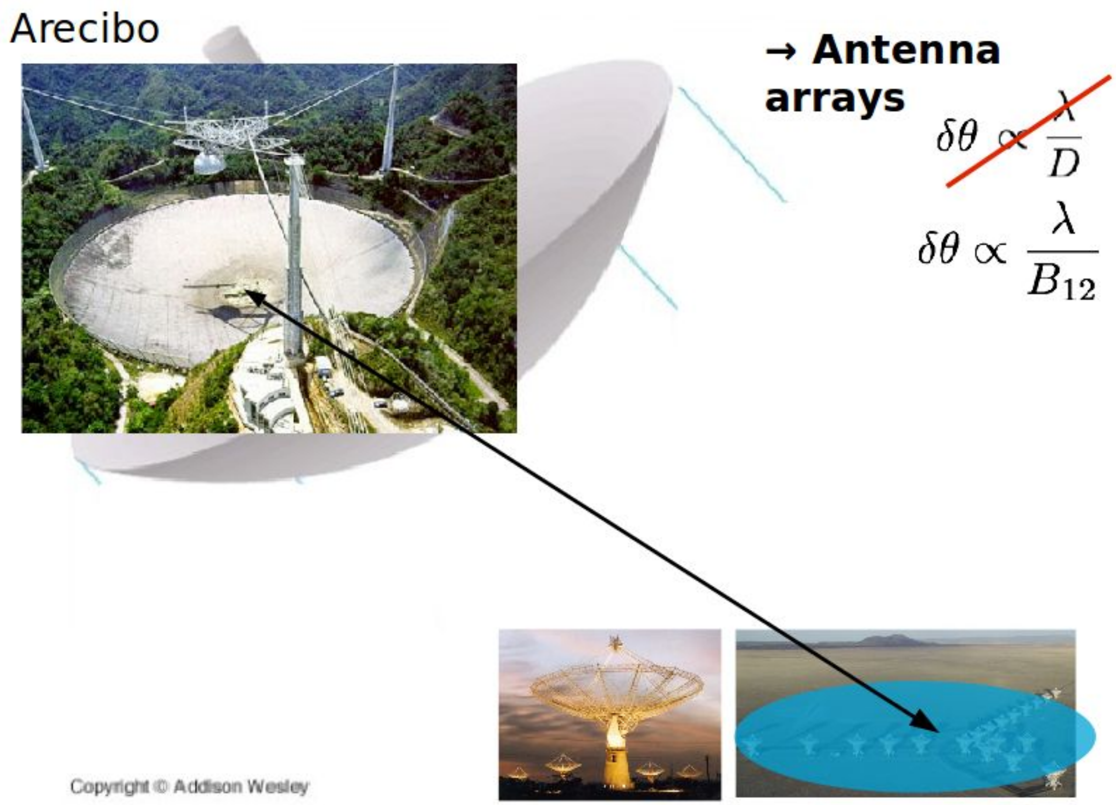
Why bother with interferometry?



$$\cancel{\delta\theta \propto \frac{\lambda}{D}}$$

$$\delta\theta \propto \frac{\lambda}{B_{12}}$$

Slide credit: Julien Girard

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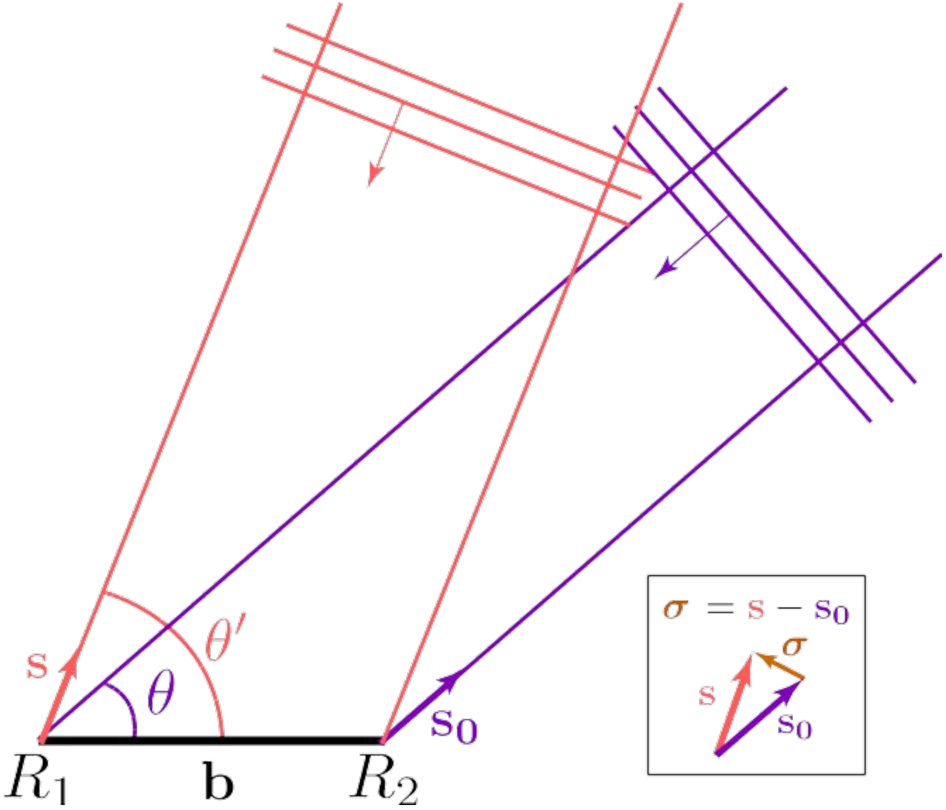
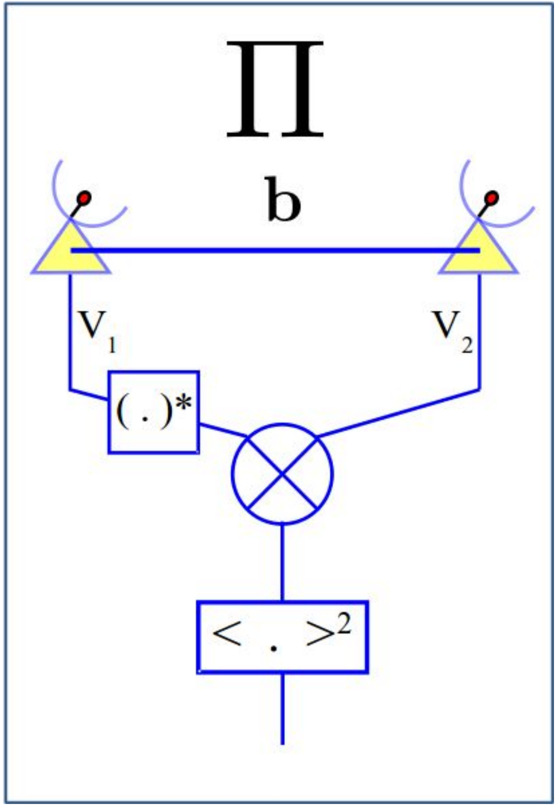
What is a visibility?





Zernike van Cittert theorem:
Visibility measures ***one Fourier mode*** of the *sky brightness distribution*!

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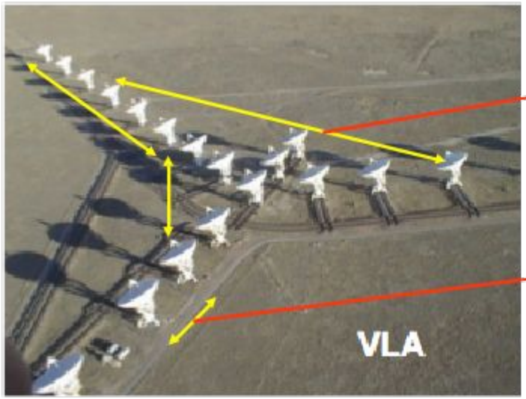
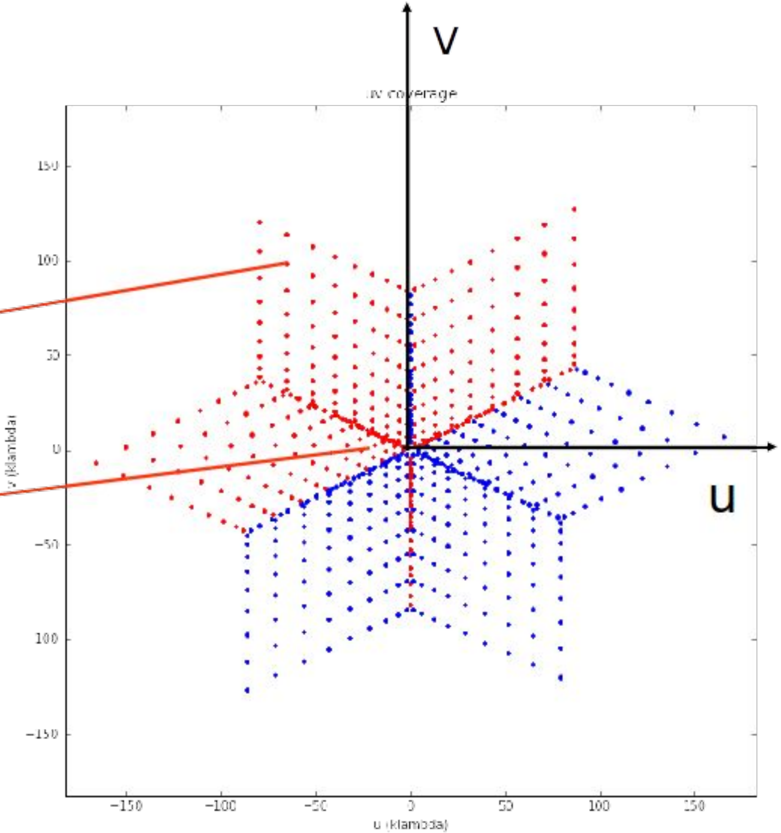
The UV-plane

Visibilities are Fourier modes:
they live in Fourier space





Radio astronomers refer to it
as *uv*-space: the denser the better!

What do we measure?

Slide credit:
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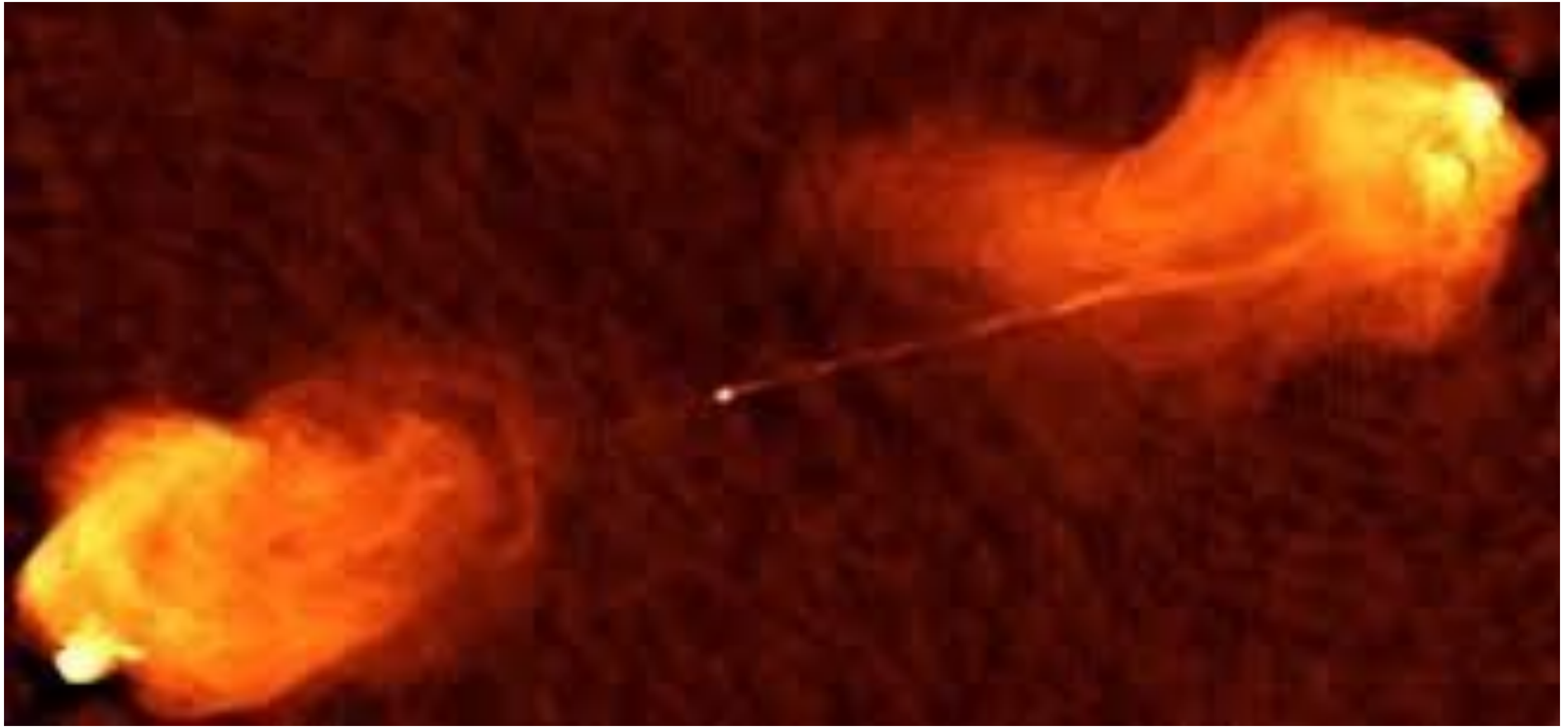

What do we measure?

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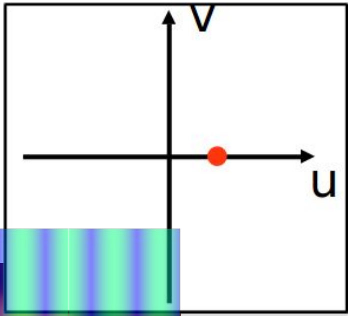
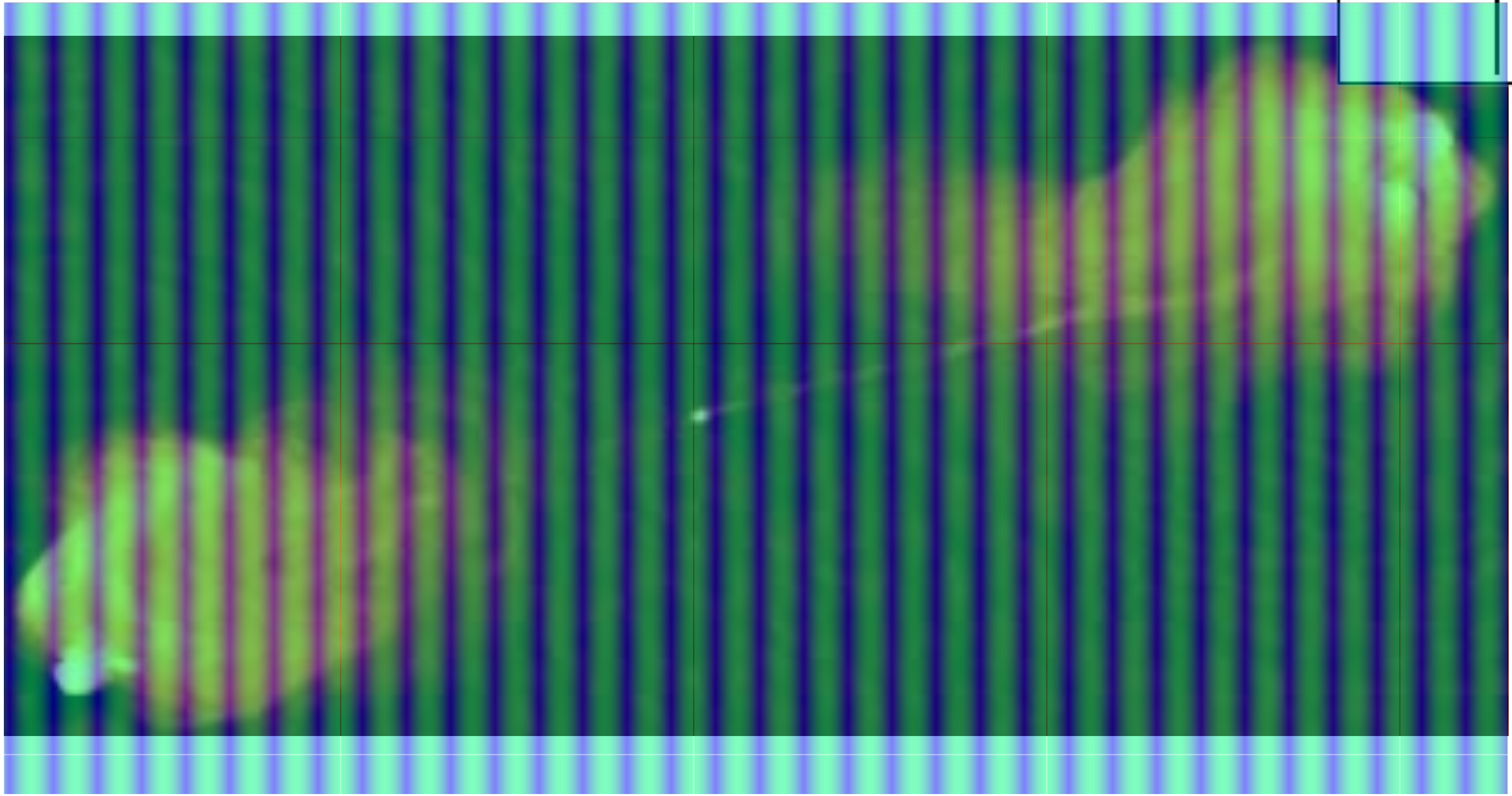
UV plane:

$\mathbf{b}_{\mathrm{proj}}$





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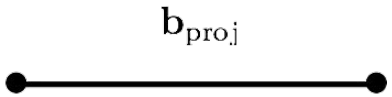
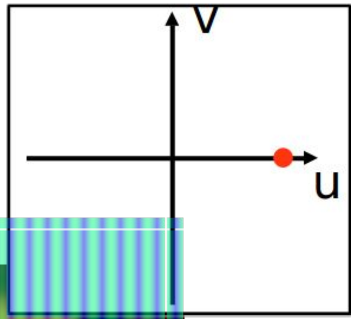
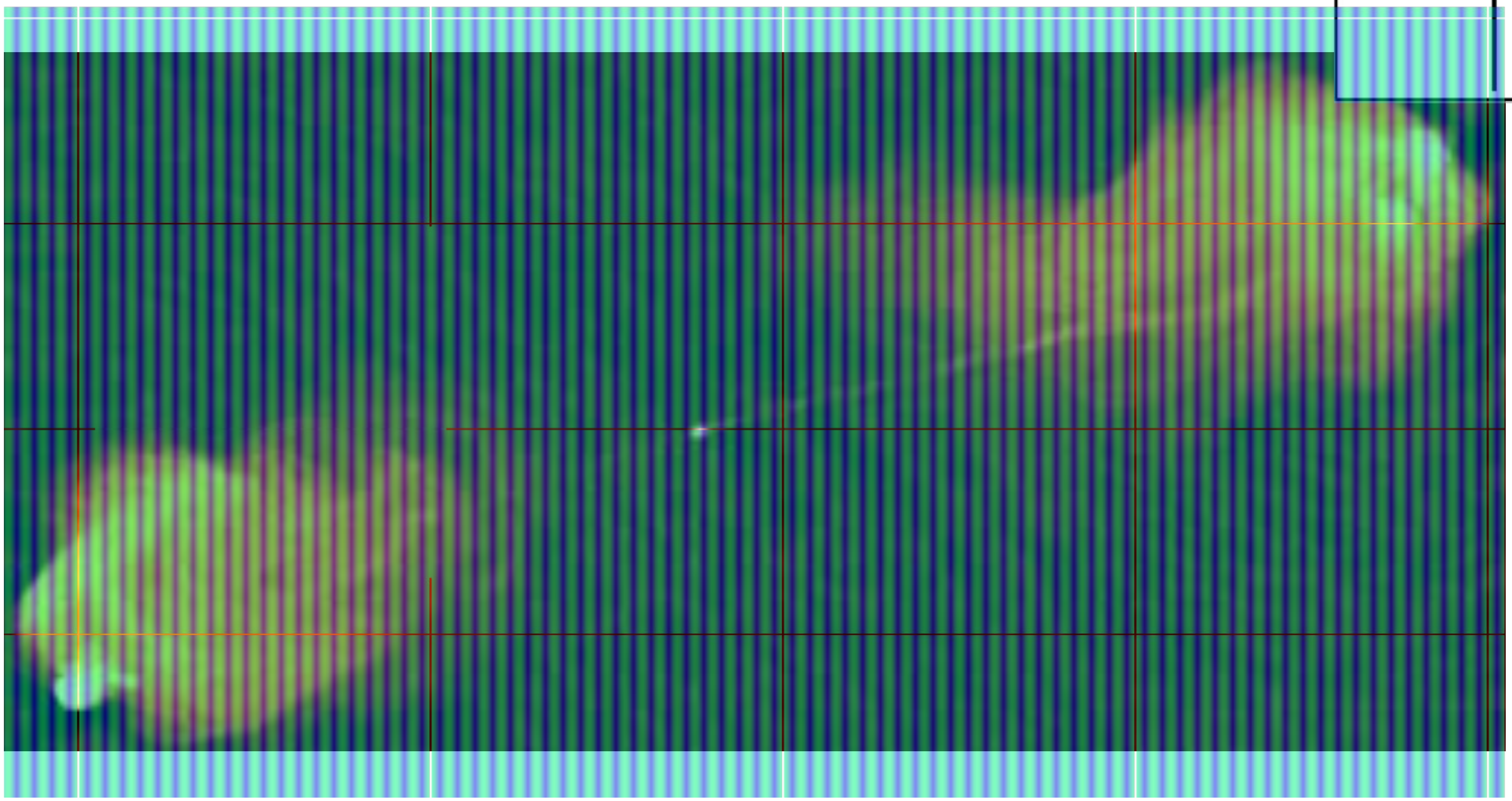


UV plane:





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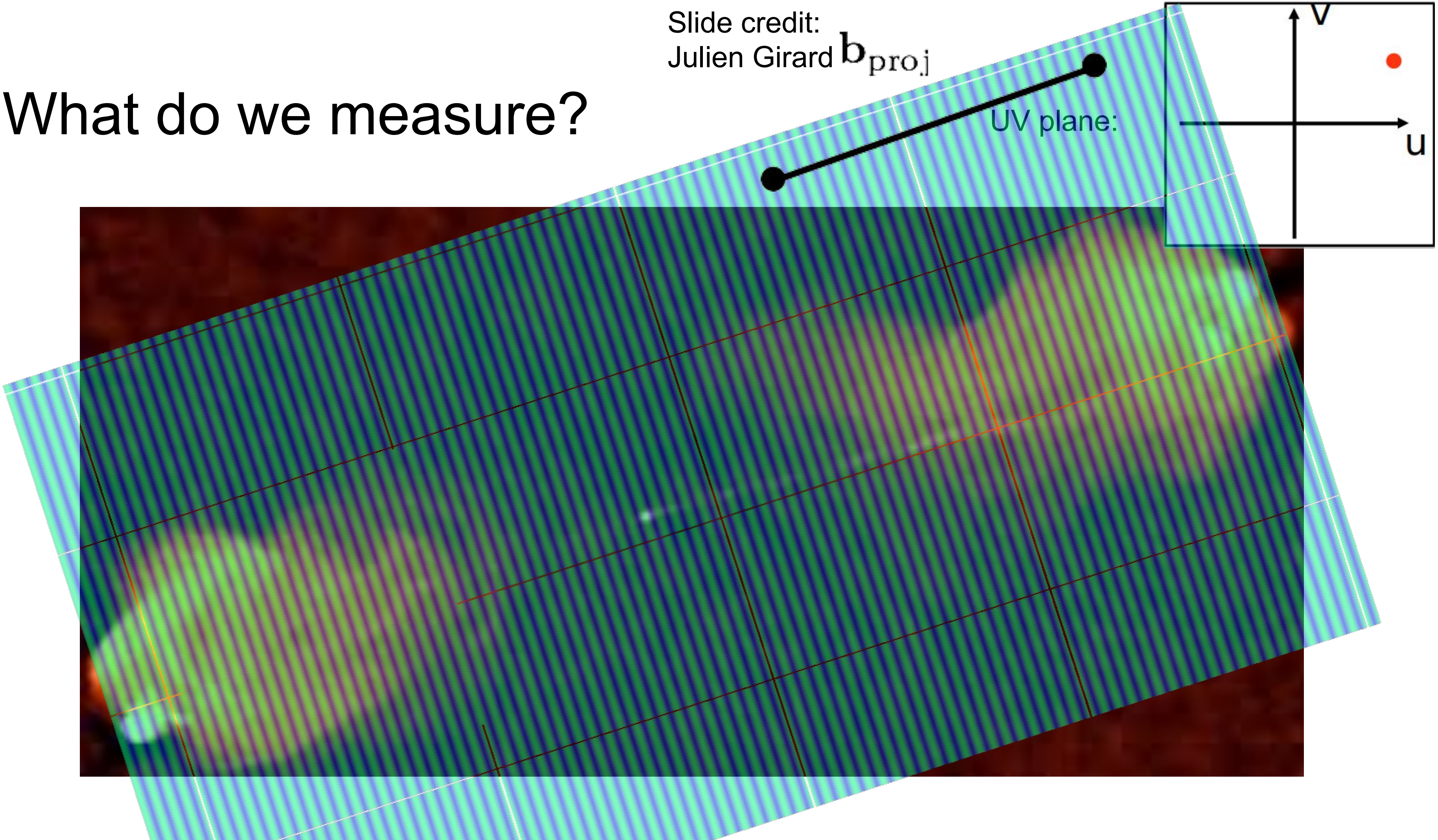

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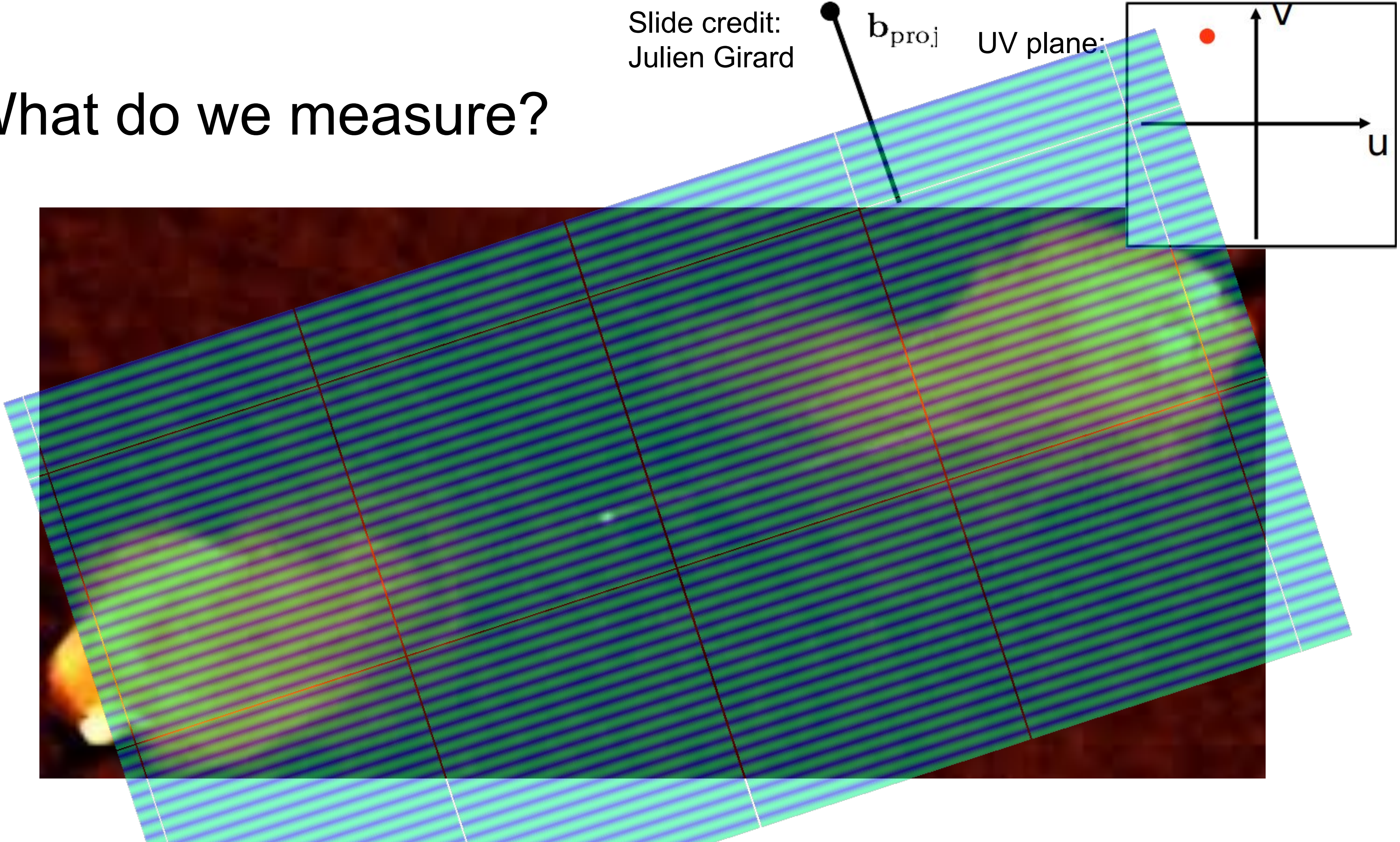

Imaging: few UV-points, bad PSF!

Slide credit:
Rick Perley, NRAO

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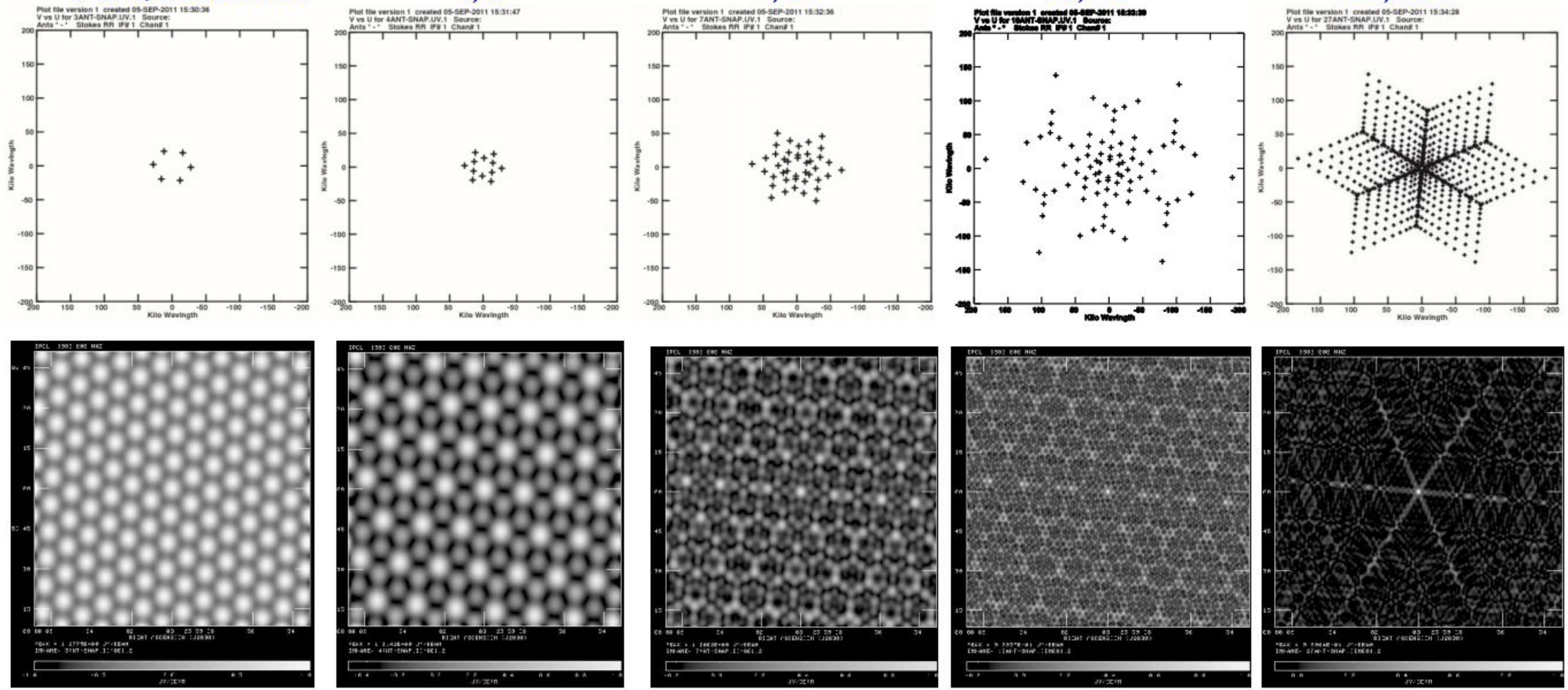

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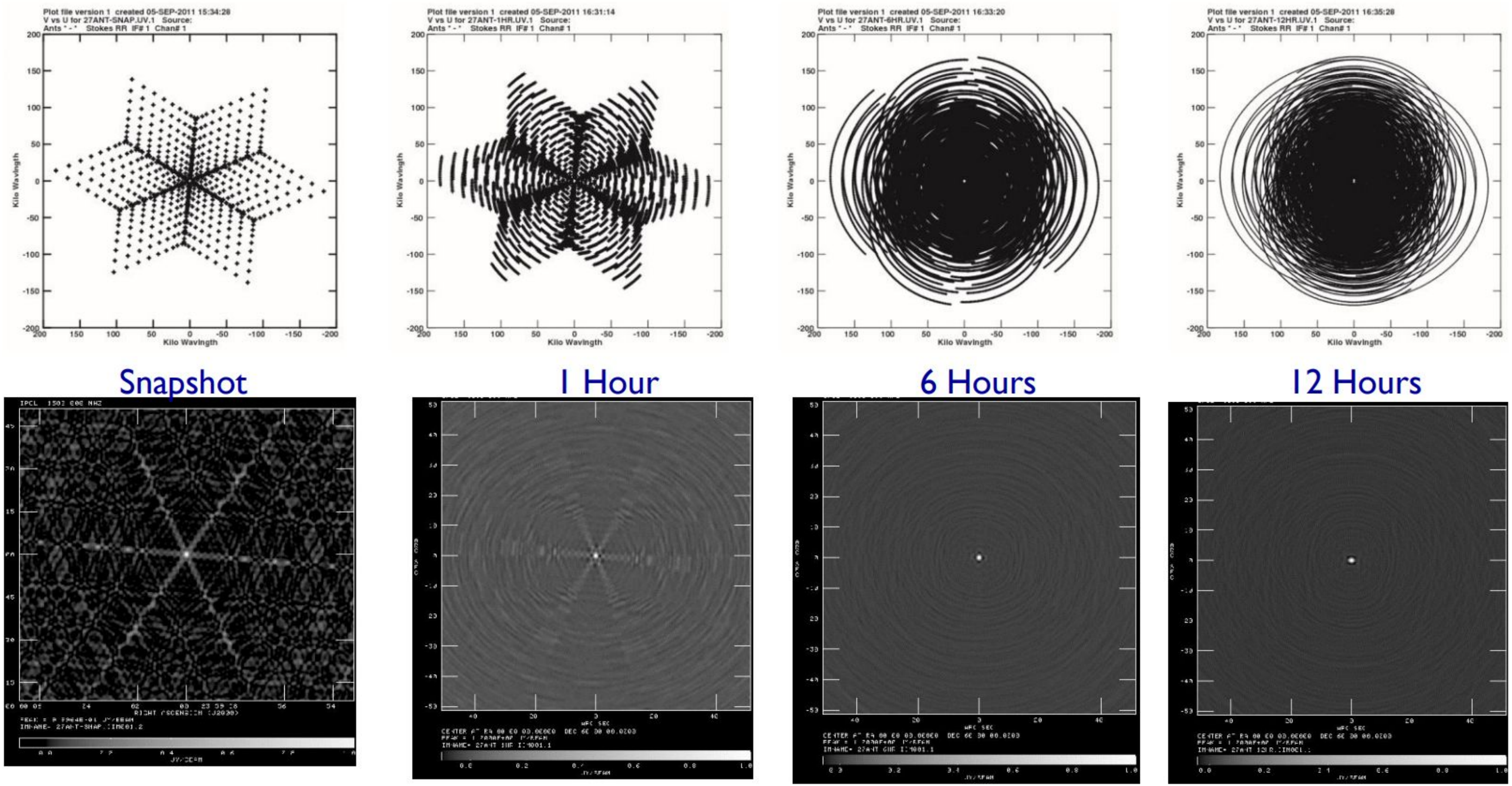

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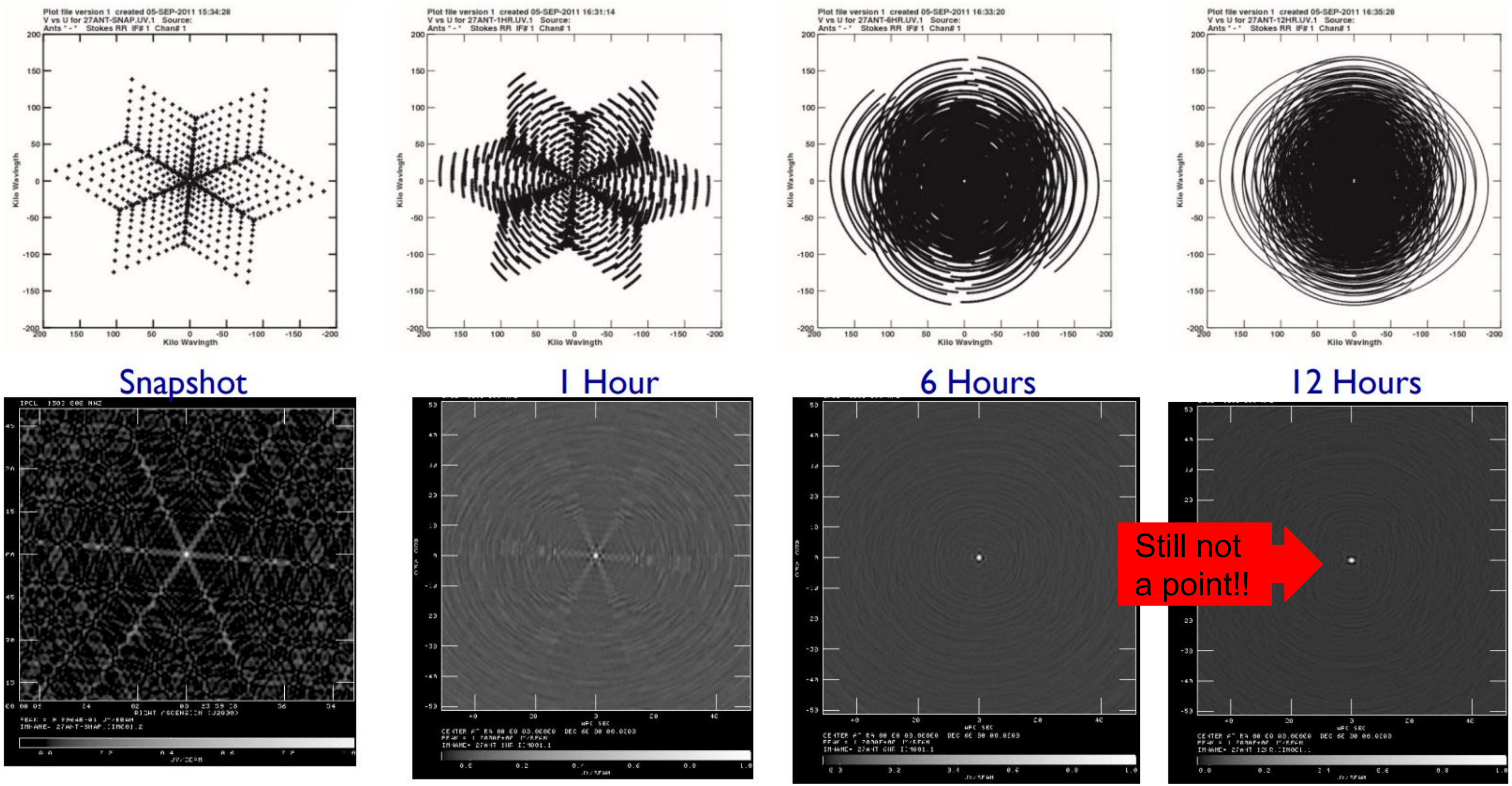
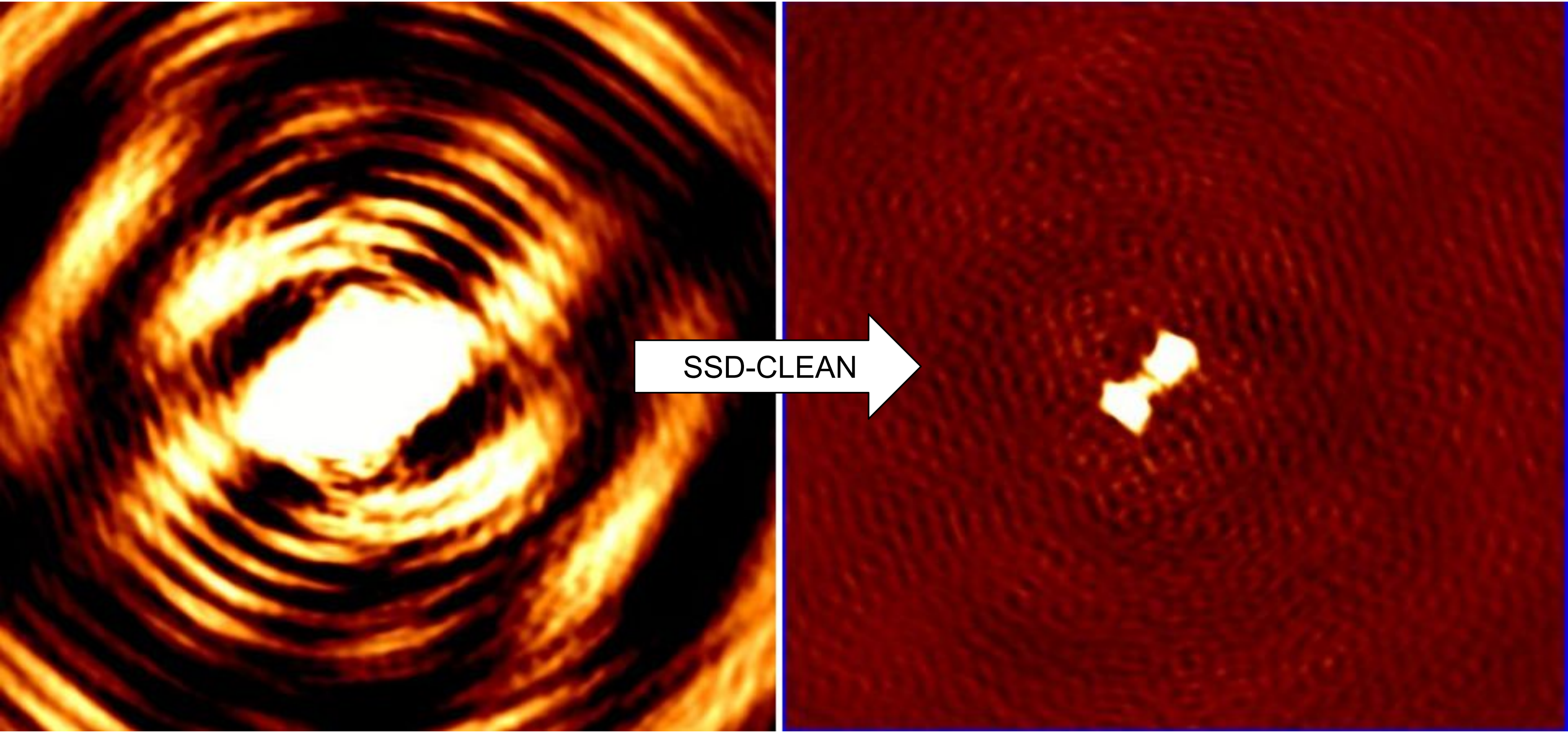


Many ways of removing PSF: CLEAN most common



Calibration - the RIME

The RIME

Measurements are voltages - not physical flux!

To correct, modern approach is Radio Interferometer’s Measurement Equation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

which implies assuming that measured voltage is linear function of sky signal. All above are 2x2 complex-valued matrices: calibration consists of **solving for $\mathbf{J}_{sp}$**.

The RIME

Fourier sampling with baseline *pq*, direction s

Measurements are voltages - not physical flux!

Noise

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Brightness in direction *s*

which implies assuming that measured voltage[illegible]ction of sky signal. All above are 2x2 complex-valued matrices: calibration consists of **solving for $\mathbf{J}_{sp}$**.

The RIME

Measurements are voltages - no

Direction-dependent effects acting on antennas *p* and *q*

Noise

To correct, modern approach is Radic nterferometer's easuremen quation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}^H_{sq} \mathbf{E}^H_{sq} \right) \mathbf{G}^H_q + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}^H_{sq} + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

Brightness in direction *s*

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The RIME

Direction-independent effects acting on antennas p and q

Measurements are voltage - not physical flux!

Noise

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$= \mathbf{G}_p \mathbf{E}_{sp} \mathbf{K}_{sp}$

Noise

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(cf. Smirnov 2011 and associated papers)

Brightness in direction *s*

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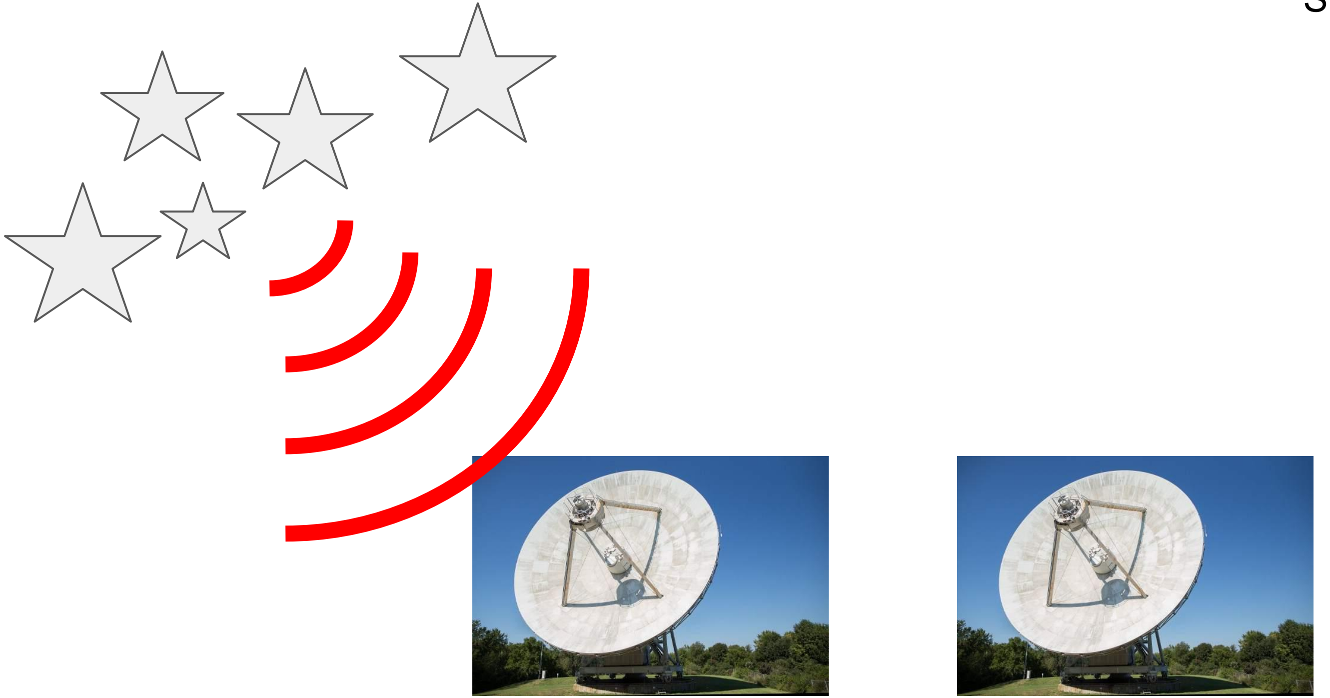
The inverse problem of interferometry: the "truth"

Signal from entire sky is measured

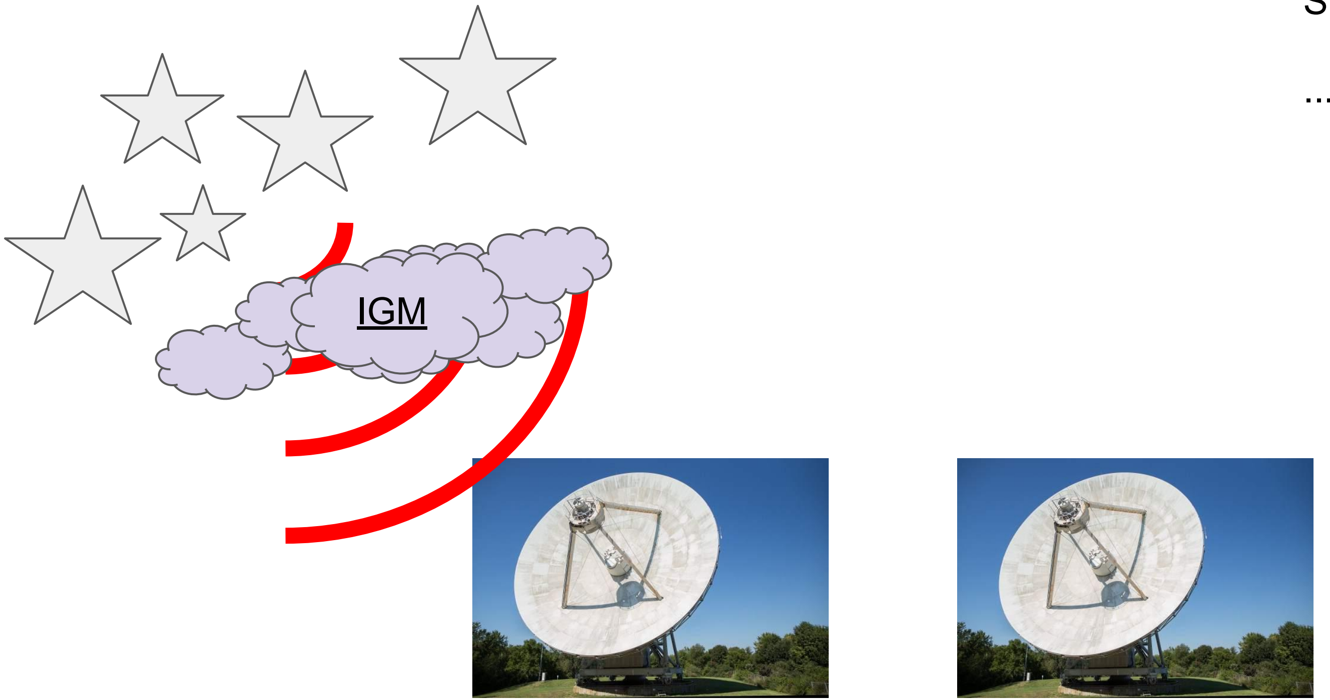


North Liberty telescope image source: https://www.thegazette.com/2013/10/08/north-liberty-telescope-peers-into-deep-space

The inverse problem of interferometry: the "truth"

Signal from entire sky is measured

...affected by IGM…



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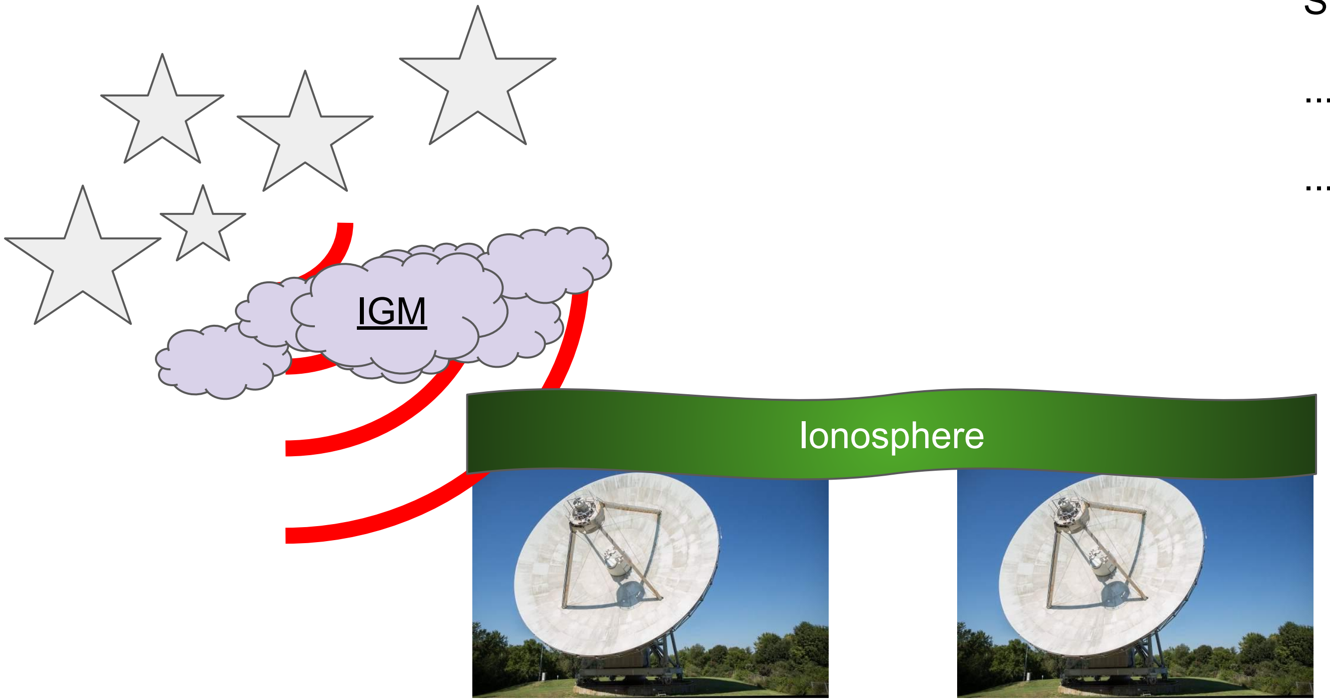

Signal from entire sky is measured

...affected by <u>IGM</u>…

...the impact of the <u>Ionosphere</u>…

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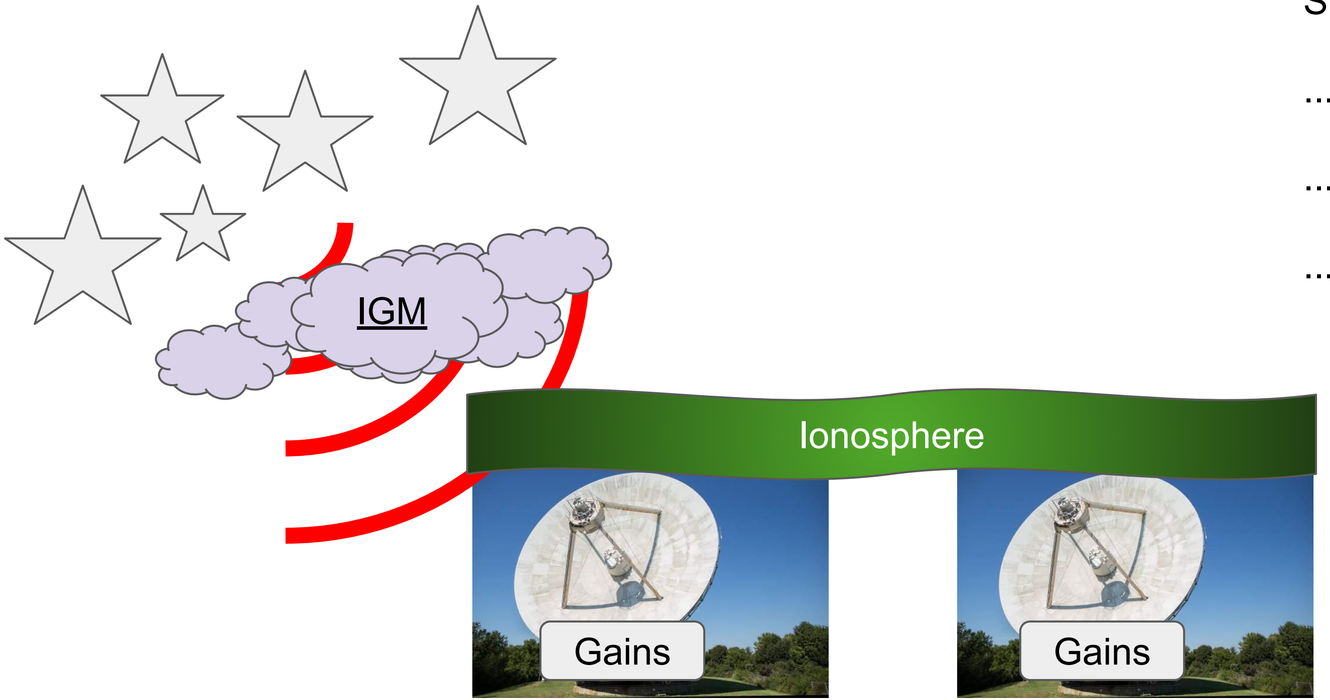

Signal from entire sky is measured

...affected by IGM…

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...the antenna gains…

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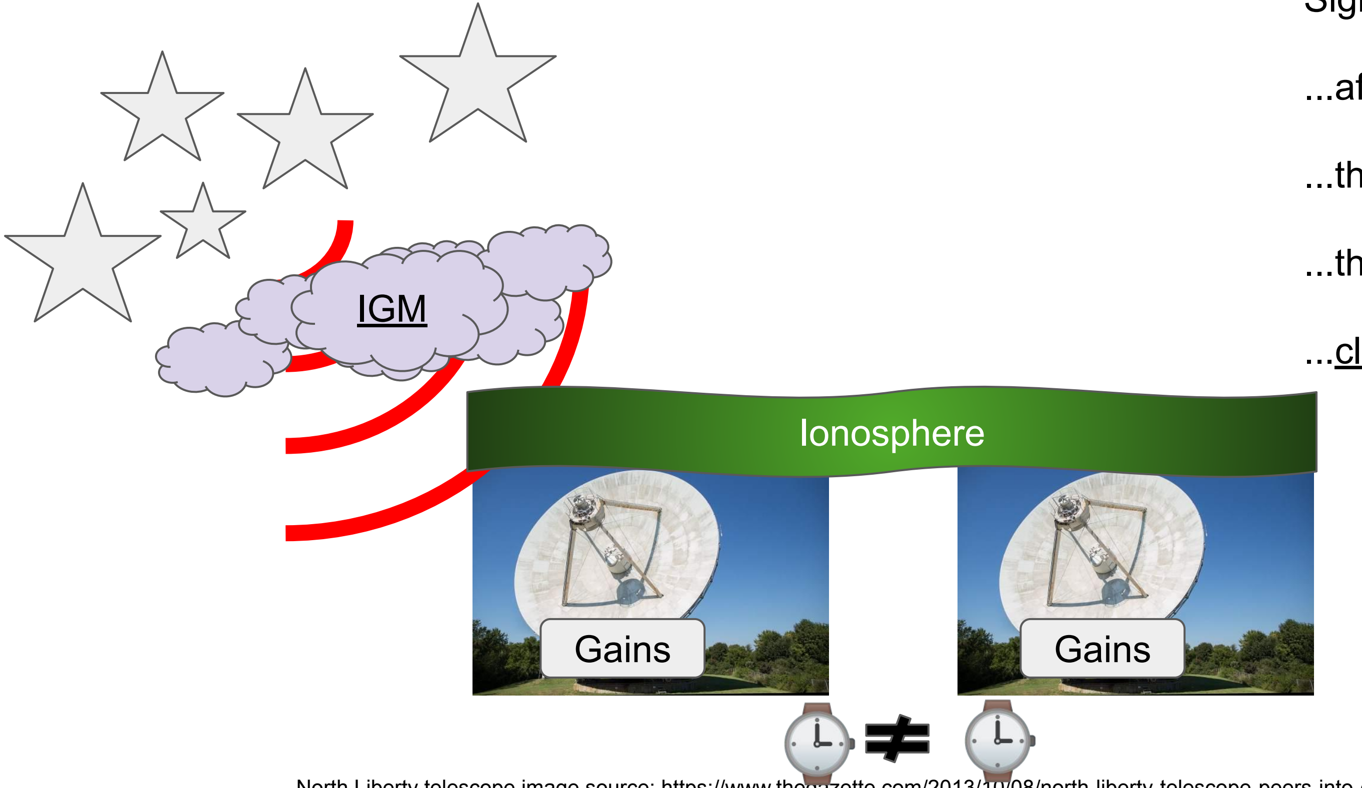

Signal from entire sky is measured

...affected by IGM…

...the impact of the Ionosphere…

...the antenna gains…

...clock errors between stations...

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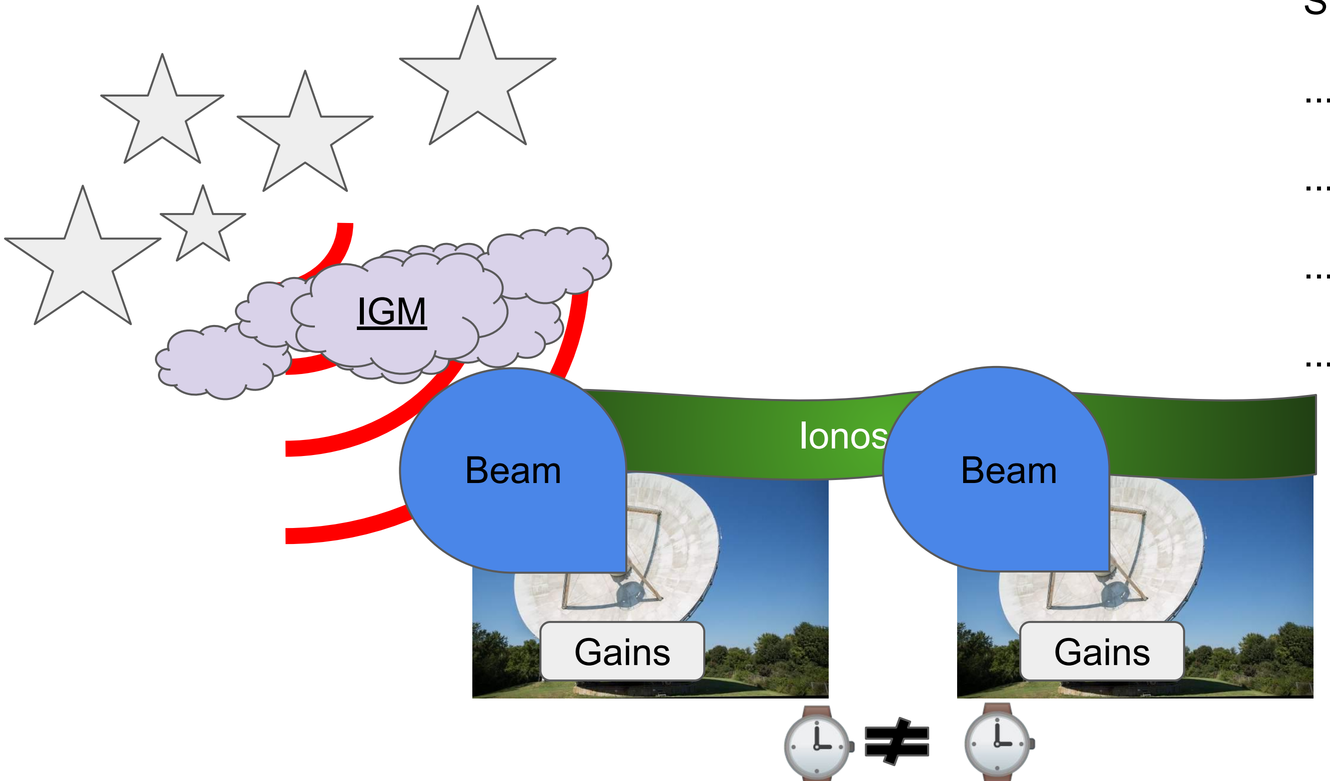

Signal from entire sky is measured

...affected by IGM…

...the impact of the Ionosphere…

...the antenna gains…

...clock errors between stations…

...antenna beam…

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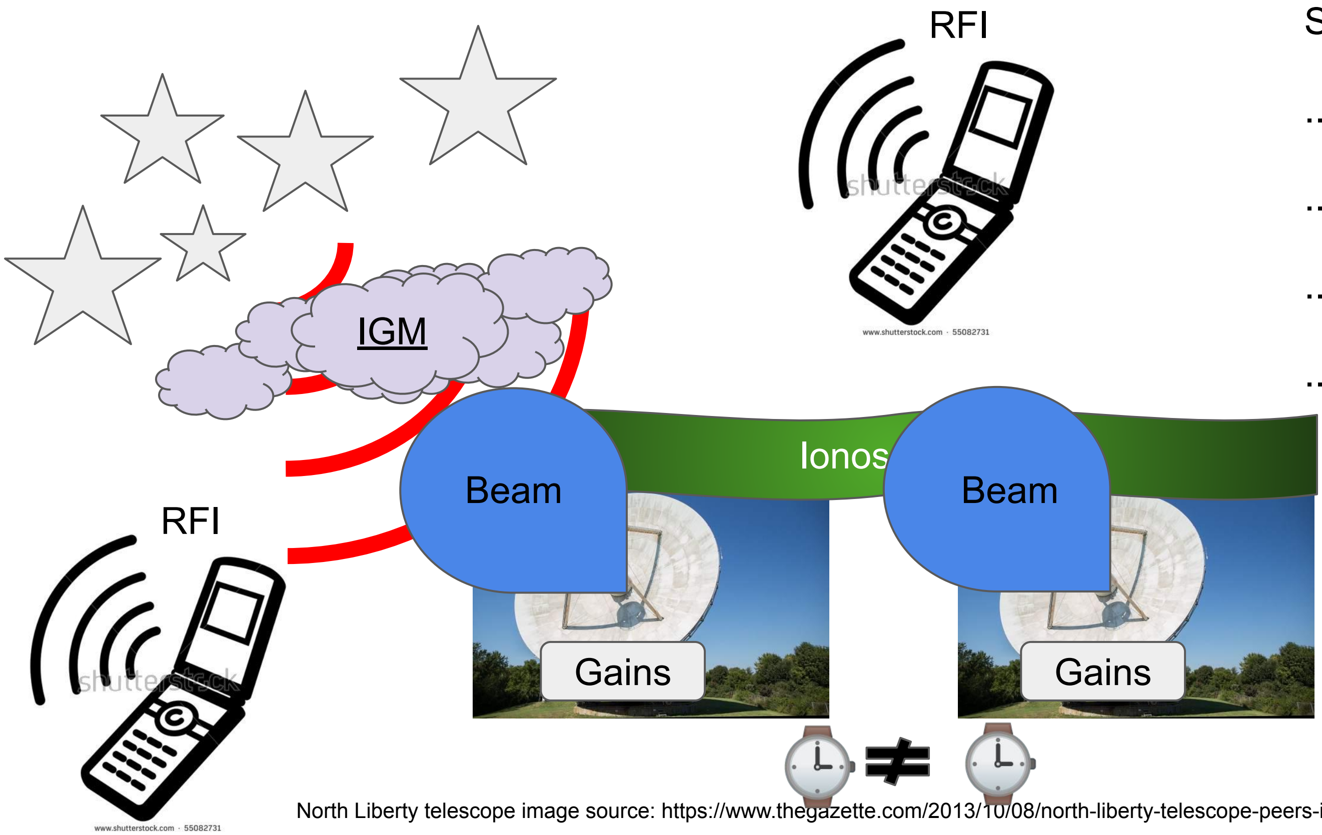

Signal from entire sky is measured

...affected by IGM…

...the impact of the Ionosphere…

...the antenna gains…

...clock errors between stations…

...antenna beam…

...radio frequency interference…

...and more besides……..

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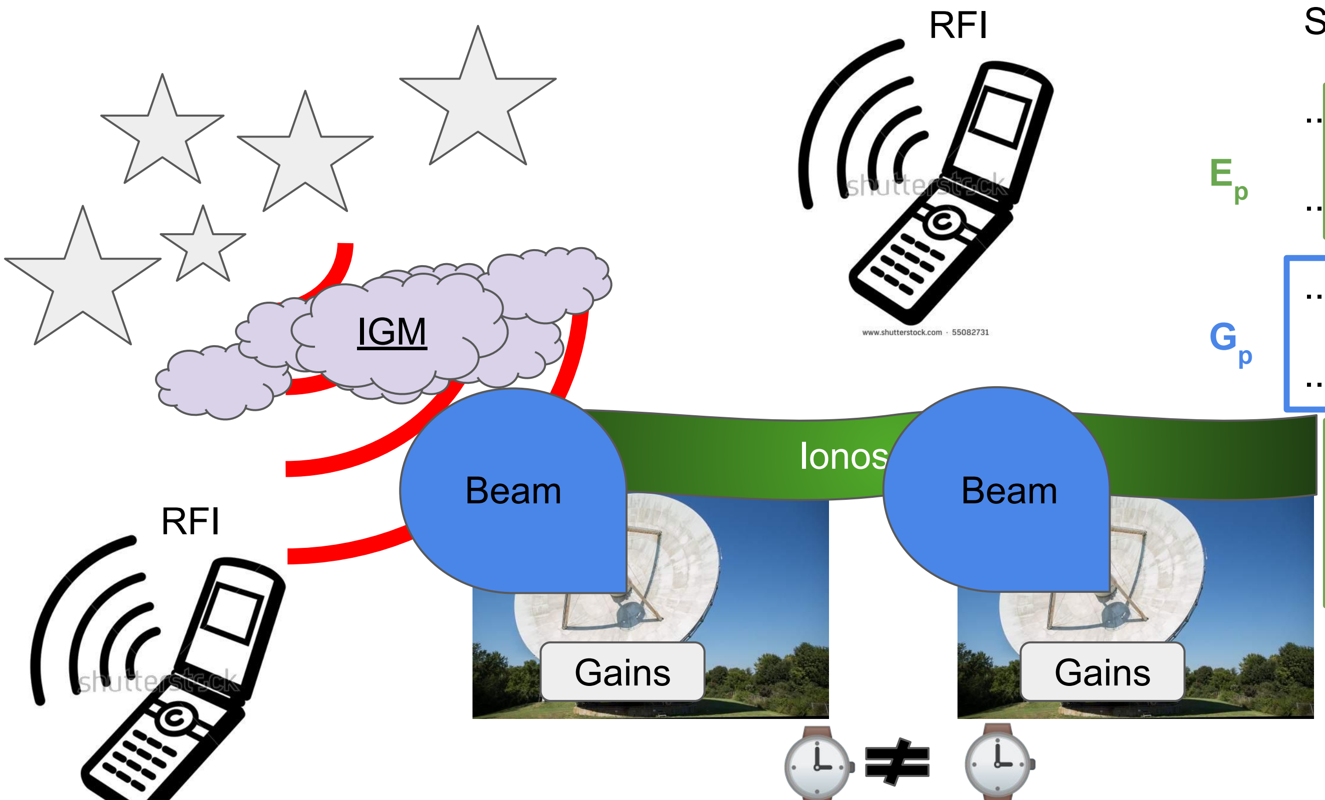
The inverse problem of interferometry: the "truth"



Signal from entire sky is measured

E_p

...affected by IGM…

...the impact of the Ionosphere…

G_p

...the antenna gains…

...clock errors between stations…

...antenna beam…

...radio frequency interference …

...and more besides……..

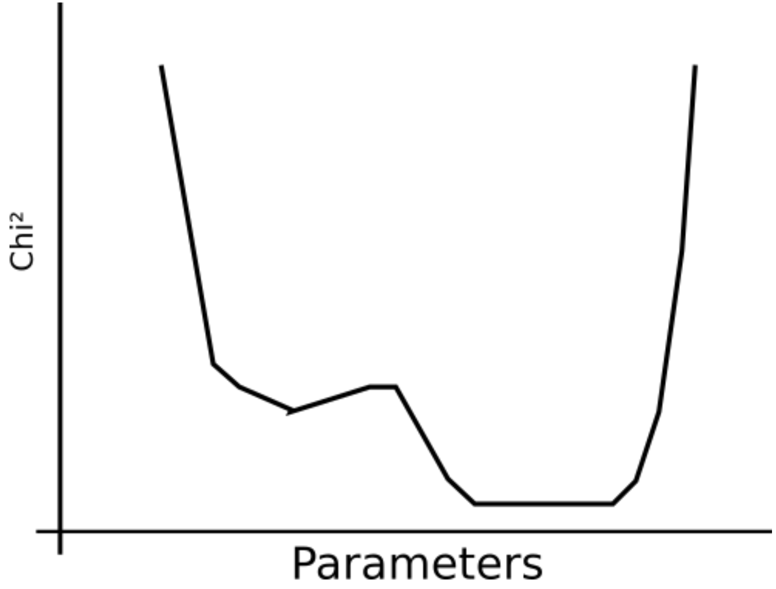
E_p

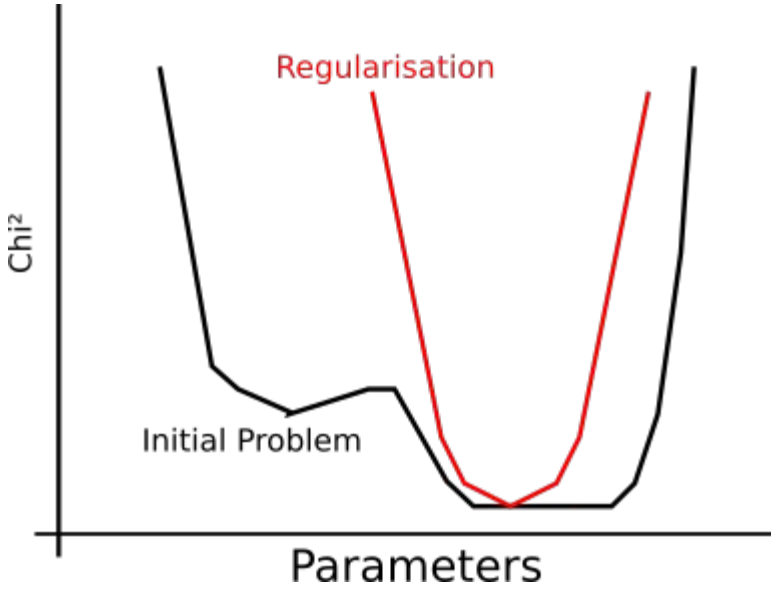
North Liberty telescope image source: https://www.thegazette.com/2013/10/08/north-liberty-telescope-peers-into-deep-space

The full inverse problem of interferometry

1. Measure visibilities
2. Calibrate - go from measurement units to physical units. This requires a good model of the sky: incompleteness of a sky model will give rise to calibration artefacts, which will pollute images made with that calibration.
3. Deconvolve - because interferometric measurements are sparse, mapping measurements to the sky is limited by a very nasty PSF. Mitigating this issue, thereby improving our model of the sky at a given frequency, requires deconvolution as a result.
4. The above are thus linked - this link is formalised in the RIME. Inverting this general problem - solving for both model and calibration solution - is the inverse problem of interferometry.

The inverse problem of interferometry - Conditioning & Regularisation





Inverse problem: not convex, poorly-conditioned ⇒ **regularise**

Generational Calibration

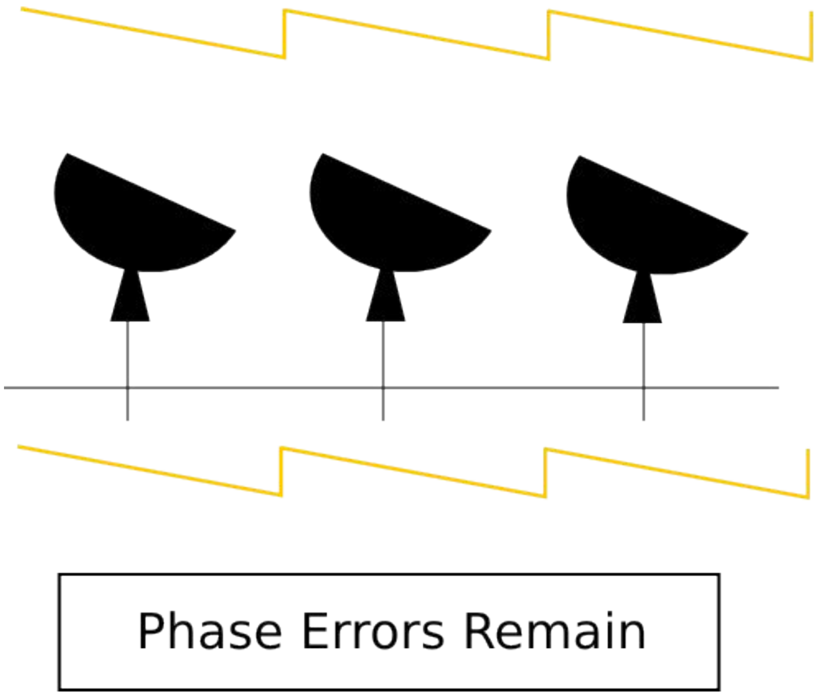
Very(!) roughly:

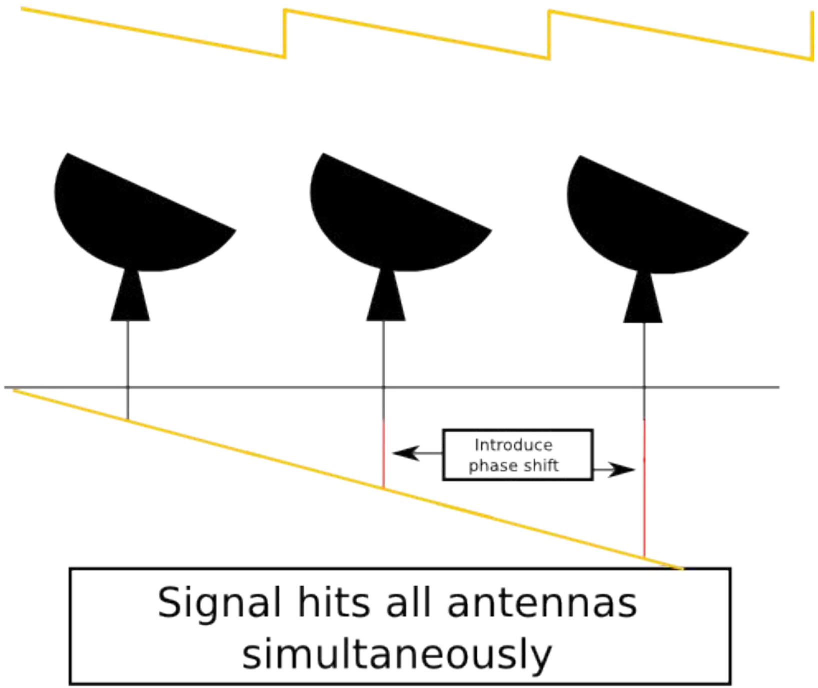
- 1GC, in the words of Jan Noordam, is comparing the signal of each baseline to the signal from a known source (the calibrator).

- 2GC is self-calibration: post-processing adaptive optics.

- 3GC is the above with direction-dependent effects taken into account.

2GC: Self-calibration as Adaptive Optics





3GC - Direction-Dependent Self-Calibration

For new generations of radio interferometers, such as LOFAR, we wish to image wide fields of view over which the direction-dependent effects in the RIME will not be constant.

This can be achieved in a few ways - here we will only discuss faceting, where we group the sources in batches over a small area, over which we assume that the direction-dependent effects are constant.

In effect, in the equation below, we now assign each source *s* to some facet Ω, and sum over the set of Ω rather than of sources. This means we assume that the DDEs are constant
over each facet.

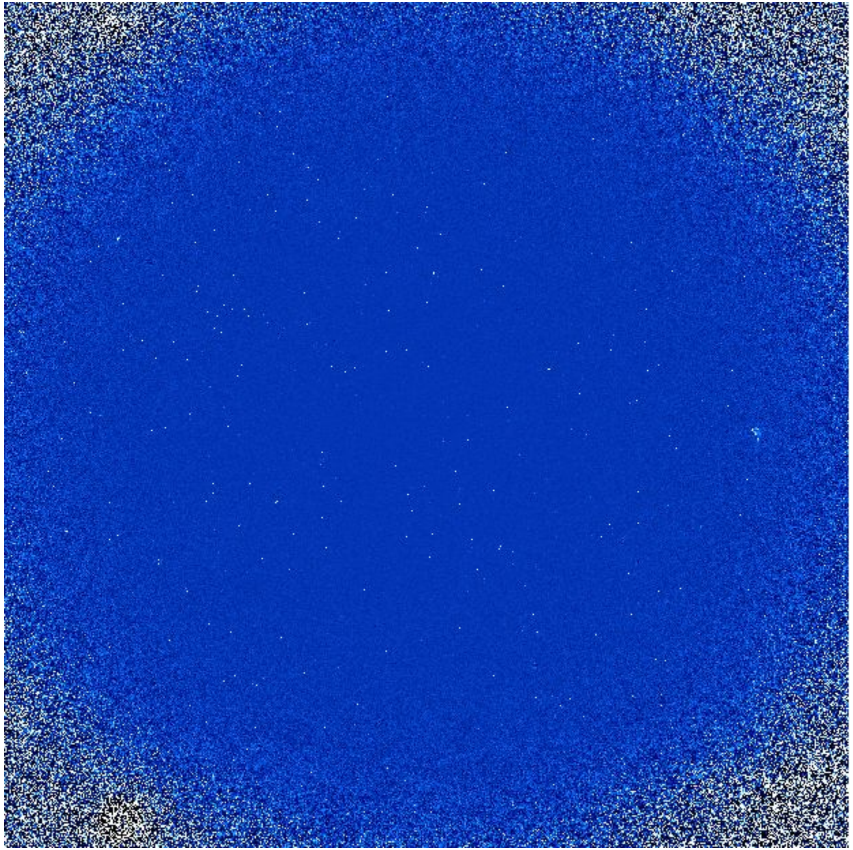
Direction-dependent effects acting on antennas *p* and *q*

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

Facet-based direction-dependent calibration

Building Facets - Voronoi Tesselation

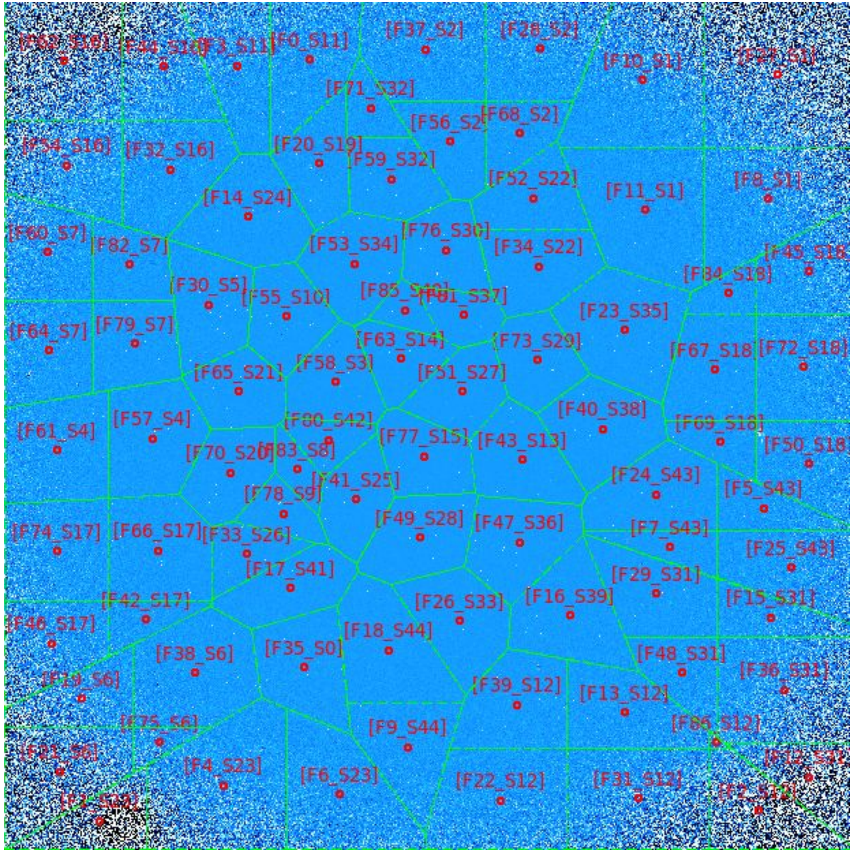
We must divide the field shown to the right into various facets, ideally each the same size, and each with enough signal to allow for calibration in that direction.



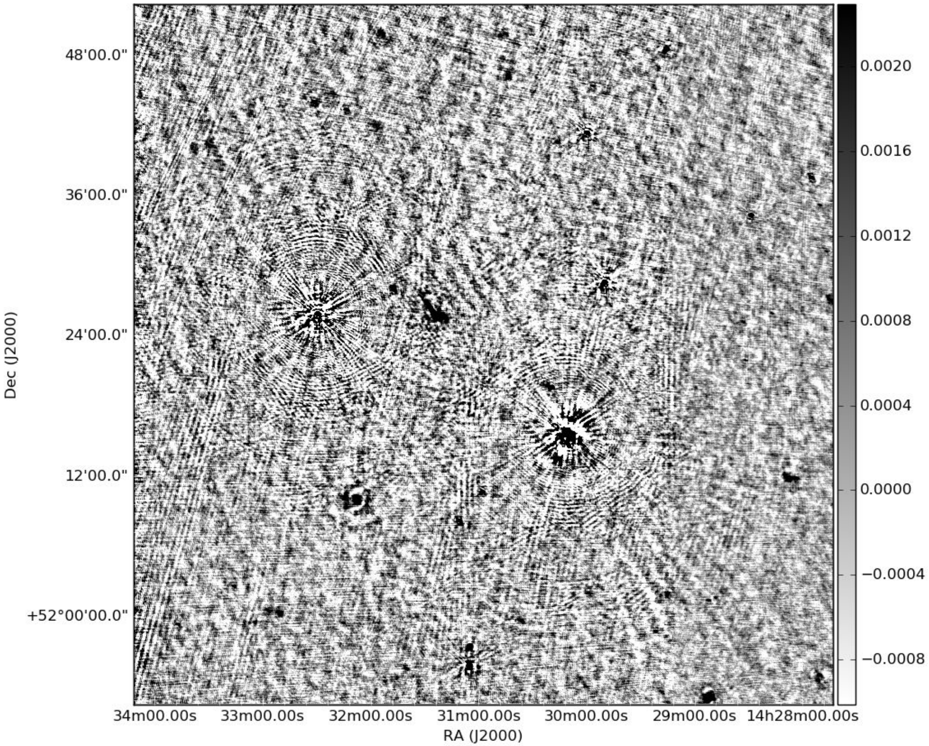
Building Facets - Voronoi Tesselation

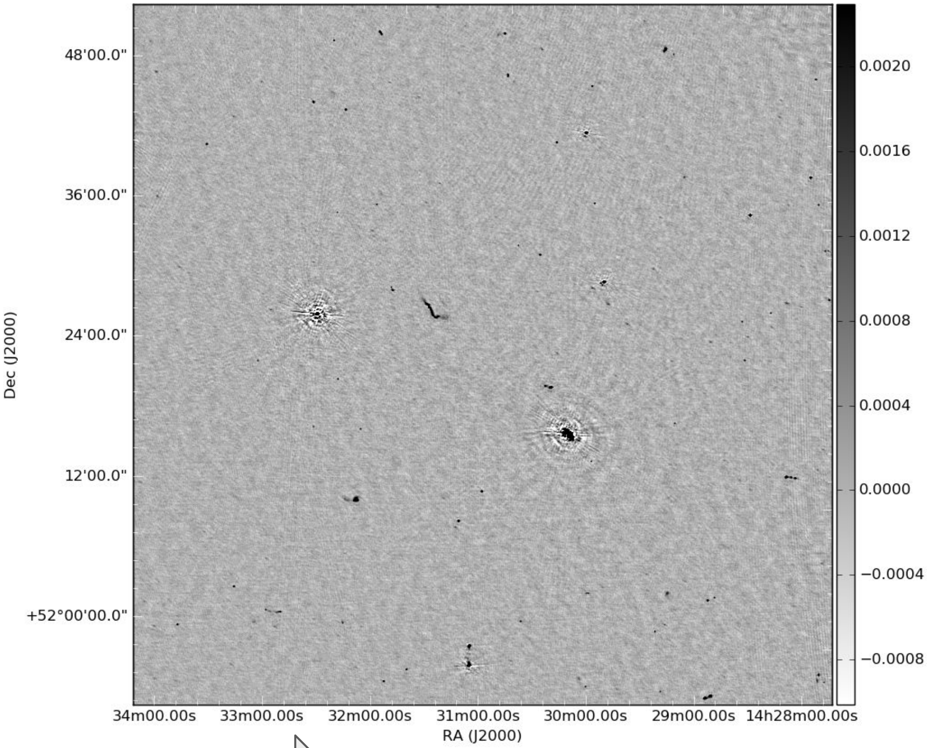
We must divide the field shown to the right into various facets, ideally each the same size, and each with enough signal to allow for calibration in that direction.

In practice, this is done through Voronoi tesselation, with results shown to the right.



Impact of Direction-Dependent Calibration





Direction-dependent Calibration

Impact of Direction-Dependent Calibration